Explaining the Investment Boom of the 1990s

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Abstract

Real equipment investment in the United States has boomed in recent years, led by
soaring investment in computers. We find that traditional aggregate econometric mod-
els completely fail to capture the magnitude of this recent growth—mainly because
these models neglect to address two features that are crucial (and unique) to the cur-
rent investment boom. First, the pace at which firms replace depreciated capital has
increased. Second, investment has been more sensitive to the cost of capital. We docu-
ment that these two features stem from the special behavior of investment in computers
and therefore propose a disaggregated approach. This produces an econometric model
that successfully explains the 1990s equipment investment boom.

1 Introduction

The behavior of equipment investment in the current U.S. expansion has been remarkable. Growth in real equipment investment over the period 1992-98 averaged 11.2 percent per year, exceeding all other seven-year intervals in the post-War era.[1] This development has been of great macroeconomic importance: The investment boom has underpinned the continuing strength of U.S. aggregate demand and has probably also had important supply-side effects, perhaps playing a role in the unusual late-cycle acceleration in labor productivity.

In this paper, we examine whether existing time series models can explain the astounding behavior of equipment investment in the 1990s. We demonstrate that they cannot. Although we examine the traditional, accelerator-style models that previous investment "horserace" studies have found best fit the data, we find that they completely fail to capture the magnitude of the 1990s investment boom.[2] We show that the models' breakdown stems from an important element of investment growth in the 1990s—the surge in real investment in *computing equipment*. Our analysis of the behavior of computer investment reveals two features that, though crucial to the investment boom of the 1990s, are ignored by standard aggregate models. We demonstrate that a *dis*aggregated approach, which models investment in computing and non-computing equipment separately, successfully explains the behavior of investment in the 1990s.

The first feature that we document is the sharp increase in the average rate of depreciation in the 1990s. Most econometric models assume a constant depreciation rate and thus a stable relationship between the change in the capital stock and the level of investment. Since the optimal capital stock is a function of the level of output and the cost of capital, this also implies a stable link between investment and changes in output and the cost of capital. However, the increasing rate of depreciation in the 1990s broke this link: Firms needed to invest more to sustain a given level of the capital stock. We show that the increase in the depreciation rate was due to a shift in the composition of capital towards computers,

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[1] All figures in this paper refer to 1992-based National Income statistics rather than the 1996-based figures published in October 1999. The econometric models in our paper use capital stock data, and revised capital stocks will not be published until Spring 2000.

[2] For earlier evaluations of competing investment models, see Clark (1979) and Bernanke, Bohn, and Reiss (1988). Oliner, Sichel, and Rudebusch (1995) compare accelerator-style models with models based on Euler equations or Tobin's Q.

which depreciate more rapidly than other types of equipment. Aggregate models do not capture this phenomenon, because, by definition, they ignore compositional mix-shifts.

The second feature that we examine is the role of the cost of capital. The rising average depreciation rate suggests the need to separately model net investment in computing and non-computing equipment. Doing so reveals an important pattern. Computer investment is very sensitive to the cost of capital, far more so than investment in non-computing equipment. As a result, rapid declines in computer prices played a crucial role in generating the investment boom of the 1990s. This result contrasts sharply with most of the empirical literature on aggregate investment, which typically finds very little response to cost variables. We provide a plausible explanation for the different estimates of cost-of-capital elasticities that we observe: Firms respond more to shocks perceived as permanent than to those perceived as transitory, and shocks to computer prices usually result from technological innovations that are unlikely to be reversed.

We conclude that the special behavior of equipment investment in the 1990s resulted from the substantial impact of rapid computer price declines on capital accumulation, and the consequent need for higher rates of replacement investment. A simple disaggregated approach, which separately models net and gross investment for computing and non-computing equipment is capable of explaining the recent behavior of investment.

The paper is organized as follows. Section 2 reviews the traditional econometric models and documents their poor empirical performance in the 1990s. Section 3 examines the increase in the average rate of depreciation and its role in the breakdown of the conventional models. Section 4 discusses why capital accumulation may respond more to the persistent component of the cost of capital than to the less persistent component. Section 5 presents our econometric analysis and documents the performance of our approach in tracking the behavior of aggregate gross investment in the 1990s. Section 6 concludes.

2   Traditional Investment Models and Their Recent Failure

Traditional models of investment start with a theory relating the optimal frictionless capital stock, K_t^*, to the production technology and factor prices. If firms could costlessly adjust the capital stock, they would always set $K_t = K_t^*$. However, the sluggish behavior of the capital stock suggests that there are costs associated with adjustment. The traditional neo-Keynesian investment models used simple *ad hoc* specifications of the effects of adjustment costs, the most common being the partial adjustment approach, which assumed that firms move part of the way towards their optimal frictionless stock each period. Formulating this relationship in terms of the logarithm of the capital stock and using lower case letters to denote the log of variables, the partial adjustment equation is

$$\Delta k = (1 - \lambda)\left(k_t^* - k_{t-1}\right) \tag{1}$$

which can be re-written as:

$$k_t = \lambda k_{t-1} + (1 - \lambda)k_t^* \tag{2}$$

Applying repeated substitution to equation (2) gives an equivalent representation for the capital stock, this time as an infinite distributed lag function of past k_t^*'s:

$$k_t = \sum_{r=0}^{\infty}(1 - \lambda)\lambda^r k_{t-r}^* = \sum_{r=0}^{\infty}\gamma_r k_{t-r}^* \tag{3}$$

This has been turned into an empirical investment equation by taking the following steps. First, the infinite distributed lag suggested by the partial adjustment theory is replaced with a finite approximation, usually about 8 to 12 quarters. Second, the equation is differenced to turn it into a net investment equation:

$$\Delta k_t = \sum_{r=0}^{N}\gamma_r \Delta k_{t-r}^* + \epsilon_t \tag{4}$$

Of course, if the capital stock adjustment equation is correctly specified, then this differencing step is not necessary. However, the traditional literature largely pre-dated cointegration methods and used stationarity-inducing transformations as a precaution against spurious regressions. In our empirical work, we will examine this issue formally.

This equation is operationalized by assuming a form for k_t^*. Specifying a CES production function, K_t^* is proportional to $\frac{Y_t}{C_t^\sigma}$, where Y_t is output, C_t is the cost of capital, and σ is

the elasticity of substitution between capital and labor. Taking logs of K_t^* we get

$$\Delta k_t = \sum_{r=0}^{N} \gamma_r \Delta y_{t-r} - \sigma \sum_{r=0}^{N} \gamma_r \Delta c_{t-r} + \epsilon_t \tag{5}$$

Since $\sum_{r=0}^{\infty} \gamma_r = 1$, the sum of the coefficients on output should approximately equal one while the coefficients on the cost of capital should sum to the elasticity of substitution, σ. These sums have an intuitive interpretation since they describe the predicted long-run response of the capital stock to permanent unit shocks to output and the cost of capital.

Models of the form of equation (5) have been estimated by Bernanke, Bohn, and Reiss (1988). However, this approach, which describes the determination of the capital stock, only gives us a model of *net* investment. For macroeconomists interested in business cycle modelling and forecasting, the variable of interest is *gross* investment, which includes both the change in capital stock and the replacement of depreciated capital. Most empirical models assume a constant average rate of depreciation and estimate an equation for gross investment. In this case, approximating the log-difference of the capital stock with the growth rate, we get

$$\Delta k_t \approx \frac{\Delta K_t}{K_{t-1}} = \frac{I_t}{K_{t-1}} - \delta \tag{6}$$

where δ is the depreciation rate. This gives an equation for gross investment

$$\frac{I_t}{K_{t-1}} = \delta + \sum_{r=0}^{N} \gamma_r \Delta y_{t-r} - \sigma \sum_{r=0}^{N} \gamma_r \Delta c_{t-r} + \epsilon_t \tag{7}$$

We will label this regression the "traditional model". Note that this approach estimates the depreciation rate as the intercept in the $\frac{I_t}{K_{t-1}}$ regression.

Previous empirical implementations of this model, estimated on data prior to the 1990s, have found that it provides a fairly good description of the cyclical behavior of investment. Indeed, Oliner, Rudebusch, and Sichel (1995) have shown that models of this form provide superior forecasting performance to popular alternative specifications based on Euler equations or Tobin's Q. This is not to say that these models are without problems. For instance, despite microeconomic evidence that the elasticity of substitution is close to one, regressions usually reveal a small and often insignificant role for the cost of capital. Indeed, comparisons of forecasting power have often favored the pure accelerator formulation ($\sigma = 0$) over models including the cost of capital. Summarizing these results, Chirinko's comprehensive 1993 survey concluded that "on balance, the response of investment to prices tends to be quite small and unimportant relative to quantity variables."

Our estimation of equation (7) confirms these previous results, revealing a small long-run cost-of-capital elasticity of -0.34. (Our data are described in Appendix A.) However, Figure 1, which shows the in-sample fit, reveals a far more serious problem. The model fails completely to capture the 1990s' increase in investment relative to the capital stock. After 1991, the model underpredicts by larger and larger amounts, with these residuals principally offset by large negative residuals over the early part of the sample.[3] By 1997:4, the actual level of investment relative to the capital stock is 7 percentage points higher than can be explained by the model; this translates into a 31 percent error on the level of investment. A Chow test for parameter stability confirms that the model has gone off track in the 1990s, with the null hypothesis of stable coefficients resoundingly rejected.[4] In the rest of the paper, we explore the reasons for the traditional model's complete breakdown in the 1990s.

3   The Unstable Aggregate Depreciation Rate

The most obvious simplifying assumption made in the derivation of the traditional model is that the average rate of depreciation is constant. We can easily check the validity of this assumption by solving for the aggregate depreciation rate obtained from re-arranging the perpetual inventory equation $(K_t = (1 - \delta_t) K_{t-1} + I_t)$ to get:

$$\delta_t = \frac{I_t - \Delta K_t}{K_{t-1}} \tag{8}$$

Figure 2 shows this series as the solid line (rather obscurely labeled "Using Chain-Weight Investment and Capital", for reasons that will become apparent in a moment). It shows that the aggregate depreciation rate has not been constant, but has increased substantially in recent years, rising from 0.13 in 1989 to 0.16 in 1997.

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[3]This is also true for popular alternative versions of this equation such as the Jorgenson "neoclassical" model and an augmented version that includes cash flow. Models based on Tobin's Q, although predicting strong investment over the last few years of our sample, also do not track the behavior of $\frac{I_t}{K_{t-1}}$ in the 1990s particulaly well.

[4]The figure also reveals a problem reported in previous horserace papers. Even when residuals are small during the middle part of our sample, they tend to be positively autocorrelated. We believe this is due to the finite-lag approximation to the true infinite-lag capital stock adjustment formula, equation (3). When λ is large (and empirical estimates suggest it is), this approximation will omit autocorrelated terms. Our regressions in Section 5 are not based on the finite-lag approximation.

The main cause of this uptrend is straightforward. Different types of equipment depreciate at different rates and Oliner (1989, 1994) has shown that computers depreciate significantly faster than other types of equipment. The National Income and Product Accounts (NIPA) capital stocks used in our analysis are constructed under the assumption of separate, constant, depreciation rates for each of 27 underlying equipment categories, with the depreciation rate for computers taken directly from Oliner's research.[5] Thus, it should come as no surprise that the recent explosion in computer investment has led to an increase in the average rate of depreciation for total equipment. Before we move on to discuss the implications of a varying pace of depreciation for econometric modelling, we need to note a surprising pattern in our calculated series for the aggregate depreciation rate.

3.1 A Depreciation Puzzle

While we had expected that the high rates of computer investment would have raised the aggregate depreciation rate in the 1990s, we were surprised to find that this was not just a recent phenomenon but rather an acceleration of a long-running trend. In fact, our calculated series for the aggregate depreciation rate *doubles* over 1965-1997. The magnitude of this apparent mix-shift seems very large, particularly as it suggests that variations in the average rate of depreciation have had an important effect on aggregate gross investment throughout the past 30 years, something not found by previous researchers. The solution to this puzzle turns out to be a change in the NIPA methodology for constructing real aggregates.

Since 1996, all NIPA real expenditure aggregates, including real GDP, have been derived using a Fisher chain-aggregation methodology.[6] Since 1997, real capital stock aggregates have been constructed using the same methodology. Rather than aggregating all quantities according to their base-year prices, as in the traditional Laspeyres index, the growth rate of a chained aggregate reflects a mix of old and new prices. Given a series of quantities and prices for n goods, $q_i(t)$ and $p_i(t)$, the gross growth rate for the Fisher chain-aggregate quantity is defined as:

$$G(t) = \sqrt{\frac{\sum_{i=1}^{n} p_i(t) q_i(t)}{\sum_{i=1}^{n} p_i(t) q_i(t-1)} \frac{\sum_{i=1}^{n} p_i(t-1) q_i(t)}{\sum_{i=1}^{n} p_i(t-1) q_i(t-1)}} \tag{9}$$

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[5]See Katz and Herman (1997) for a description of the NIPA stocks.

[6]See Landefeld and Parker (1997) for a discussion of this methodology.

In the base-year (1992 in our data), all price indexes are set equal to one and the level of each aggregate is set equal to its nominal value. For all subsequent and previous years, the real level series are simply "chained" forward and backwards using the Fisher-aggregation growth rates. For NIPA aggregate real equipment investment and stocks, the Fisher chain procedure aggregates 27 component series.

This chain aggregation procedure helps to reduce biases due to valuing goods at prices that become irrelevant once we move away from the base year. However, a complexity it introduces is that *the level of the constructed real aggregate is no longer the additive sum of its real components*, with this lack of additivity being most evident when there are large relative price shifts within a bundle of goods (as is the case with the equipment bundle because of the substantial declines in the price of computing equipment). This non-additivity invalidates the calculation of the aggregate depreciation rate. To illustrate, consider the following simple example.

There are two types of capital, A and B. Suppose now that the aggregate capital stock is constructed according to the traditional Laspeyres fixed-weight formula. In this case, the real aggregates for investment and the capital stock are the simple sum of their real components: $I^{FW} = I^A + I^B$ and $K^{FW} = K^A + K^B$. It is easy to use this fact to show that the aggregate depreciation rate is a weighted average of the two underlying depreciation rates, with the weights given by the real quantities for the two stocks:

$$\delta_t^{FW} = \frac{I_t^{FW} - \Delta K_t^{FW}}{K_{t-1}^{FW}} = \delta^A \left(\frac{K_{t-1}^A}{K_{t-1}^A + K_{t-1}^B} \right) + \delta^B \left(\frac{K_{t-1}^B}{K_{t-1}^B + K_{t-1}^A} \right) \qquad (10)$$

Note, however, that the strict additivity of the fixed-weight formula was necessary to obtain this weighted-average expression for the aggregate depreciation rate. Once this additivity breaks down, only in the base year can we interpret the depreciation rate calculated from equation (8) as a weighted average; this is because in the base year all real series are equal to their nominal counterparts and so for this year additivity does hold. In fact, as we show in Appendix B, moving away from the base year, the aggregate depreciation rate calculated from equation (8) with chain-weighted data differs systematically from a weighted-average depreciation rate, displaying a long-run upward trend even in the absence of mix shifts towards faster depreciating equipment.

The explanation for this result is fairly subtle; a full derivation is available in Appendix B. However, the intuition is as follows. The growth rate of a chain-weighted aggregate

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effectively equals a weighted-average of the growth rates of its components, where the weights are given by the components' nominal shares. It turns out that when real investment in one type of capital grows faster than others because its relative price is declining, then the nominal share of this type of capital in investment will be higher than its nominal share in the capital stock, implying that the aggregate for real investment grows faster than the aggregate for the real capital stock. As a result, the ratio of the level of real aggregate investment to the level of the real aggregate capital stock, will trend upwards. Hence, the series calculated from equation (8) will also trend upwards.

To demonstrate the effect on the aggregate depreciation rate of the change in aggregation methodology, we constructed fixed-weight aggregates for equipment investment and the equipment capital stock by adding up the underlying real series for the 27 equipment investment categories. We then calculated a depreciation rate for these fixed weight series, exactly as in equation (10). This fixed-weight depreciation rate series is shown as the thick dashed line on Figure 2 (labeled "Using Fixed-Weight Investment and Capital"). It rises steeply over the past few years but increases only very slightly prior to the 1990s, remaining for most of the sample in the range of 0.13, the value most commonly used in studies that construct equipment stocks from a constant aggregate depreciation rate. In contrast, the corresponding series for chain-weighted data climbs steadily from the mid-1960s on. Thus, although the increasing aggregate depreciation rate in the 1990s mainly reflects a composition shift, the uptrend evident prior to the 1990s is mainly an artifact of chain aggregation.[7]

Since mix-shifts towards equipment-types that are faster depreciating and declining in relative price can explain the substantial rise over time in our perpetual-inventory estimate of the aggregate depreciation rate, an obvious question is whether removing computing equipment (which depreciates rapidly and has the largest price declines) will result in a stable depreciation rate. The final series on Figure 2, the thin dashed line (labeled "Non-Computing Equipment, Chain-Weight Investment and Capital"), tells us that the answer is: Almost. This series was calculated by applying equation (8) to newly-calculated chain-aggregates for investment and capital stock for all equipment except computers, and it shows a very slow and modest upcreep over time.[8]

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[7]Because previous research in this field used the old fixed-weight data, this explains why other researchers did not note this curious pattern.

[8]Appendix A describes how we calculated these aggregates for non-computing equipment.

3.2 Implications for Aggregate Investment Modelling

Returning to the recent failure of the traditional model, we have seen that the actual depreciation rate required to convert aggregate net investment ($\Delta k_t = \frac{I_t}{K_{t-1}} - \delta_t$) into $\frac{I_t}{K_{t-1}}$ is not a constant that can be proxied by the intercept, as is assumed when we directly estimate equation (7), but in fact has been rising rapidly in recent years. If our aim is a stable econometric model, then a solution to this problem is to instead directly model the behavior of net investment by estimating equation (5). Figure 3 illustrates how this step radically improves in-sample fit. It compares the residuals from our estimation of the gross investment ($\frac{I_t}{K_{t-1}}$) model, equation (7), with the residuals from estimation of the net investment ($\frac{I_t}{K_{t-1}} - \delta_t$) model, equation (5). Once we do not have to account for the variations in the aggregate depreciation rate, we no longer have residuals that trend up over time and the recent net investment residuals, though still positive and relatively large, are not historically unprecedented.

This is something of a hollow victory, however, if our ultimate goal is a model of gross investment expenditures. Worse still, these aggregate models cannot explain the source of the increasing aggregate depreciation rate–the explosion in net investment in computing equipment. A complete model of gross investment expenditures in the 1990s must account for the different behavior of investment in computing and non-computing equipment. In the next section, we present an alternative to the partial adjustment model that provides intuition for such an approach by illustrating why computer prices may have a different effect on investment than other elements of the cost of capital.

4 Cost of Capital Shocks and Capital Accumulation

Empirical tests of the traditional models that we have focused on thus far find only a small role for price variables. Therefore, they imply that rapidly declining computer prices have had little impact on investment in computers. The sheer magnitude of the increase in computer investment in recent years suggests that this may be incorrect. Consider now an alternative theoretical approach, previously presented by Nickell (1979) and Kiyotaki and West (1996), that explains why computer price declines may affect capital accumulation more than other shocks.

The models we have looked at thus far rely on very simple modelling of the effects of adjustment costs. An alternative is to explicitly model the implications of adjustment costs for an optimizing firm with rational expectations. To capture only the essential features of the investment problem, we use a quadratic approximation to the underlying profit function: Changes in the capital stock and deviations from the frictionless optimal stock both lead to costs which increase according to a simple quadratic function. For a given expected path of k^*, firms choose the current capital stock to solve

$$Min \quad E_t \left[\sum_{m=0}^{\infty} \theta^m \left\{ (k_{t+m} - k_{t+m}^*)^2 + \alpha \left(k_{t+m} - k_{t+m-1} \right)^2 \right\} \right] \tag{11}$$

where θ is the firm's discount rate.

The model's first-order conditions are:

$$E_t \left[-k_{t+1} + \left(1 + \frac{1}{\theta} + \frac{1}{\alpha \theta} \right) k_t - \frac{1}{\theta} k_{t-1} - \frac{1}{\alpha \theta} k_t^* \right] = 0 \tag{12}$$

Letting L be the lag operator, F be the lead operator, and using the fact that the characteristic equation $x^2 - \left(1 + \frac{1}{\theta} + \frac{1}{\alpha \theta} \right) x + \frac{1}{\theta} = 0$ has two roots such that one root (λ) is between zero and one while the other equals $\frac{1}{\theta \lambda}$, this can be re-expressed as

$$E_t \left[- (F - \lambda) \left(F - \frac{1}{\theta \lambda} \right) Lk_t - \frac{1}{\alpha \theta} k_t^* \right] = 0$$

Implying a solution

$$k_t = \lambda k_{t-1} + \frac{\lambda}{\alpha} E_t \left[\sum_{n=0}^{\infty} (\theta \lambda)^n k_{t+n}^* \right] \tag{13}$$

An intuitive re-formulation of this equation that illustrates the model's fundamental property, comes from using the fact that $\frac{\lambda}{\alpha} = (1 - \lambda)(1 - \theta \lambda)$ (which comes from re-

arranging the characteristic equation). Making this substitution we get

$$\Delta k_t = (1 - \lambda) \left(k_t^{**} - k_{t-1} \right) \tag{14}$$

where

$$k_t^{**} = (1 - \theta\lambda) E_t \left[\sum_{n=0}^{\infty} (\theta\lambda)^n k_{t+n}^* \right] \tag{15}$$

Thus, each period, the log of the capital stock adjusts towards the moving target, k_t^{**}, which is a weighted average of expected future k_t^*'s. It can be shown that λ depends positively on α, implying that higher adjustment costs lead to a slower speed of adjustment towards k_t^{**}.[9]

The model is completed by a specification of the process for k_t^*. Profit maximization (using a generalized CES production function) will give us a first order condition: $k_t^* = \eta_t + y_t - \sigma c_t$, where y and c are as before and η_t summarizes the effects of capital-biased technological change. To give a concrete example of what η_t means, the stock of computing capital may tend to rise independently of output and the cost of capital if the structure of production changes in ways that facilitate increased usage of computers.

To implement this model empirically, we need to specify time series processes for output and the cost of capital. Let $y_t = \phi(L) y_{t-1} + \epsilon_t$ and $c_t = \pi(L) c_{t-1} + \nu_t$ where ϕ and π are m-th order distributed lag polynomials and ϵ_t and ν_t are white noise. Given these processes, we can solve for the effects of output and the cost of capital on k_t^{**} in terms of empirically observable variables by using the following formula of Hansen and Sargent (1980):

$$E_t \left[\sum_{n=0}^{\infty} (\theta\lambda)^n y_{t+n} \right] = \kappa(L) y_t \tag{16}$$

where

$$\kappa(L) = \frac{1}{1 - \phi(\theta\lambda)} \left[1 + \sum_{k=1}^{m-1} \left(\sum_{r=k+1}^{m} (\theta\lambda)^{r-k} \phi_r \right) L^k \right] \tag{17}$$

Similarly, letting

$$\mu(L) = \frac{1}{1 - \pi(\theta\lambda)} \left[1 + \sum_{k=1}^{m-1} \left(\sum_{r=k+1}^{m} (\theta\lambda)^{r-k} \pi_r \right) L^k \right] \tag{18}$$

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[9]This type of capital stock process can also be derived from more general assumptions about technology and adjustment costs: See Auerbach (1989).

the process for the capital stock is now

$$k_t = \lambda k_{t-1} + (1 - \lambda)(1 - \theta\lambda)(\kappa(L) y_t - \sigma\mu(L) c_t) + \hat{\eta}_t \qquad (19)$$

where $\hat{\eta}_t$ depends on a weighted average of current and future values of the capital-biased technological change term.

Suppose now we estimate equation (19). The technology-bias variable, $\hat{\eta}$, cannot be observed, so this ends up in the error term. Thus, our estimating equation is

$$k_t = \alpha + \lambda k_{t-1} + \sum_{i=0}^{N} \beta_i y_{t-i} + \sum_{i=0}^{N} \gamma_i c_{t-i} + u_t \qquad (20)$$

where the model predicts that

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\begin{aligned}
\beta(L) &= (1 - \lambda)(1 - \theta\lambda)\kappa(L) \\
\gamma(L) &= \sigma(1 - \lambda)(1 - \theta\lambda)\mu(L) \\
\alpha + u_t &= \hat{\eta}_t
\end{aligned}
$$

Equation (20) bears a close resemblance to the capital stock equation under partial adjustment. However, the coefficients on y and c now depend on the variables' own time-series processes and the discount rate, θ, as well as on the underlying production technology and the adjustment speed, λ. Specifically, consider the long-run elasticities with respect to y and c, defined as the sum of coefficients on these variables divided by $(1 - \lambda)$. These values depend positively on the *persistence* of the explanatory variables. In Appendix C, we show that if c is an $I(1)$ series, then $(1 - \theta\lambda)\mu(1) = 1$. But, if c is an $I(0)$ series, then this term is less than 1, and will be approximately zero if c is white noise. The reason for this result is intuitive: Firms are less likely to react to shocks to the "frictionless optimal" stock that they perceive as being temporary than to shocks perceived to be permanent.[10]

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[10]Note that these are *conditional* elasticities, not long-run impulse responses of a multiple equation system: They describe the behavior of the capital stock conditional on the paths of output and the cost of capital. This contrasts with the work of Kiyotaki and West (1996). They have also noted that this model can allow the capital stock to have different elasticities with respect to output and the cost of capital. However, their empirical implementation imposed the assumption that the cost of capital was an $I(1)$ series, thus ruling out this possibility. Their implementation of this model instead focused on long-run impulse responses of the (k, y, c) system. Their finding of smaller long-run impulse responses to shocks to c comes from their estimated process for c being a less persistent $I(1)$ process than the $I(1)$ process for output (for instance although both are $I(1)$ processes, $y_t = 1.5y_{t-1} - 0.5y_{t-2} + \epsilon_t$ implies larger impulse responses than $y_t = 0.5y_{t-1} + 0.5y_{t-2} + \epsilon_t$). It does not come from smaller *conditional* elasticities for k with respect to c than with respect to y.

In light of these results, it is informative to examine the persistence properties of the cost of capital for computing and non-computing equipment. We define the cost of capital according to the standard Hall-Jorgenson rental rate formula:

$$C_t = P_t \left(R_t + \delta - \frac{\dot{P_t}}{P_t} \right) \left(\frac{1 - ITC - \tau * DEP}{1 - \tau} \right) \tag{21}$$

where P_t is the price of capital relative to the price of output, R_t is the real interest rate, ITC is the investment tax credit, DEP is the present value of depreciation allowances per dollar invested, and τ is the marginal corporate income tax rate.

Expressed in logs, the cost of capital is the sum of two series–the relative price of capital, and the non-relative-price component, which measures the tax-adjusted gross required rate of return on investment. As Figure 4 shows, these two components affect the computer and non-computer cost of capital series in very different ways. The upper panels show that the computer cost of capital is highly non-stationary, exhibiting continuous rapid declines as a result of the remarkable pattern of falling purchase prices. The lower panels show that the relative stability of the non-computer cost of capital comes from a combination of an uneven decline in the relative price of this equipment and a choppy pattern for the non-price component.

Even looking within specific categories, the cost of capital combines components that appear to have very different persistence properties. For instance, the relative price of computers appears to be a very persistent series; the relative price of non-computing equipment seems to have a downward trend, although one that is less dominant than for computers; the non-price components for both variables seem to be relatively stable, mean-reverting series. More formal econometric characterizations of the persistence of these series, using simple autoregressions and unit root tests, confirm the intuition implied by these graphs. These tests suggest that the relative price series for both computing and non-computing equipment almost certainly have unit roots, while the non-relative price components appear more likely to be stationary series.

There are also good *economic* reasons to believe that the price and non-price components of the cost of capital have different persistence properties. The pattern of declining relative prices for equipment comes from technological innovations in the equipment-producing industries, and it seems likely that once prices have fallen as a result of innovations, these price reductions will be permanent. In contrast, real interest rates will, in the long-run,

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be related to the marginal productivity of capital, which will be a stationary variable in any general equilibrium model. Similarly the Hall-Jorgenson tax term is bounded and has tended to be mean-reverting.

To summarize, explicitly modelling the effects of adjustment costs tells us that the effect on investment of shocks to the cost of capital depends on the perceived persistence of the shocks. We have also shown that the persistence of the cost of capital varies substantially across equipment type, with the cost of capital for computers being dominated by the persistent decline in purchase prices. These results suggest using a disaggregated approach that allows different types of equipment to have different elasticities with respect to the cost of capital.

5   Econometric Modelling

5.1   Regressions

We estimated the capital stock adjustment formula, equation (20), for aggregate equipment as well as for computing and non-computing equipment. Because the proposed regressions contain nonstationary variables, we first addressed whether there is a cointegrating relationship. We ran the potential cointegrating regressions and applied Phillips-Ouliaris-Hansen tests for a unit root in the residuals. We could not reject the hypothesis that the error term has a unit root for any of the three categories. (This may be because our error term contains the biased technological change term $\hat{\eta}_t$, and it is possible that this term has a unit root.) These results indicate that the conventional approach in the "horserace" literature of differencing to avoid a spurious regression was probably well-founded.[11] We will follow this approach in estimating a differenced version of equation (20).[12]

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[11]For completeness, we also estimated our regressions in levels; the important results of this section were unchanged.

[12]Note, though, that our approach of directly estimating the capital stock adjustment equation differs from the approach of the traditional models. These models applied repeated substitution of the lagged k_t term to transform the theoretical $ARMA$ equation into an $MA(\infty)$ equation, and then approximated this equation using an an $MA(n)$ regression. However, if the adjustment cost parameter, λ, is high (and empirical estimates suggest that it is), then terms omitted in this $MA(n)$ approximation will still have large coefficients. Since these terms are probably positively autocorrelated, we believe that this accounts for the poor autocorrelation properties of the traditional models.

The results are shown in Table 1. The aggregate results (column 1) are familiar from previous empirical investment papers. The estimated λ of 0.93 implies relatively slow adjustment. The sum of the coefficients on output is significantly positive and the sum of the coefficients on the cost of capital, though negative as expected, is quite small. The long-run elasticities are shown in the bottom part of the table. For the cost of capital, this elasticity is only -0.18.

The second column of Table 1 shows this regression for computing equipment. Limited data availability requires us to estimate over a smaller sample for computing equipment (1980-97), which leads to less tightly estimated coefficients.[13] Nonetheless, this column contains an important result: *The estimated long-run elasticity of the computer capital stock with respect to the cost of capital is -1.6, nearly 9 times the estimate from the aggregate model.* Column 3 reports the results for non-computing equipment; these are similar to the aggregate regression.

According to the model in the previous section, regressors with more persistent time series processes should have higher elasticities. Thus, part of the explanation for the larger cost-of-capital elasticity for computing equipment could be that the variance for the computer cost of capital is dominated by persistent shocks (falling computer prices). Columns 4-6 examine this hypothesis and provide confirmation. For both computing and non-computing equipment, the elasticities with respect to the more persistent components of the cost of capital (the relative price terms) are larger—in the case of computers, significantly so. Moreover, the long-run investment elasticity with respect to computer prices is also statistically significantly larger than the non-computer elasticity with respect to non-computer prices.[14]

In fact, by estimating the persistence properties of the various regressors we can calculate exactly *how much* higher the elasticities on persistent regressors should be. We estimated processes for price and non-price variables for both computing and non-computing equipment, using a stationary representation for the non-price variables, and imposing the

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[13]We chose this starting data because the stock of computing equipment was very small before 1980. None of the results reported here are sensitive to the choice of sample.

[14]The results we have shown in this section are robust. Durbin's h statistics are low indicating that the regressions are free of residual autocorrelation. Specification changes (such as including a trend and adding extra lags) did not significantly alter any of our results. Furthermore, the regressions show no evidence of parameter instability in the 1990s.

assumption that the processes for the price variables are $I(1)$. Using these processes along with equations (17), (18), and (20), we find that the cross-equation restrictions implied by the model tell us that, for both computing and non-computing equipment, the conditional elasticity of the capital stock with respect to the non-price variables should equal about half the elasticity with respect to the price variables. A Wald test of these cross-equation restrictions reveals that they cannot be rejected. However, because of the relative imprecision of the estimates we are reluctant to place too much emphasis on these tests.

Our assumption that the relative price series are $I(1)$ also implies that the estimated long-run elasticities with respect to these variables should equal the elasticities of substitution for each type of capital. The implied elasticity of substitution for non-computing equipment is -0.33, in line with standard estimates from previous investment studies, although still perhaps surprisingly low. For computing equipment, the implied elasticity of substitution of -1.83 is extremely large. A possible interpretation of this result is that computer technologies are more easily substitutable for other factors.

5.2   Implications of Computer Price Measurement Error

One question about our large estimate of the elasticity of computer net investment with respect to its relative price is whether it could be affected by errors in the measurement of computer prices. The reasons to suspect that measurement error may be affecting this coefficient are twofold. First, the NIPA computer price index is a constant-quality series. This price is constructed from so-called "hedonic" price regressions, and there is certainly room for mis-specification and mis-measurement in these regressions. Second, like almost all NIPA expenditure categories, real investment in computing equipment is constructed by deflating the nominal expenditure series by the price index. Thus, any measurement error in the price index will affect both the right- and left-hand sides of our net investment regression.

While such measurement error may affect our regressions, we believe that consideration of this factor points to a price elasticity for computing equipment that is *larger* in magnitude than our estimate. This is because this type of measurement error biases the estimated long-run elasticity with respect to prices towards minus one and our estimate is -1.83. To illustrate this result, consider a simplified version of our theoretical investment equation,

without dynamics or non-price cost-of-capital terms:

$$\Delta k_t = \alpha + \beta \Delta y_t - \gamma \Delta p_t + \epsilon_t$$

Suppose now that the NIPA price, p^*, is measured with error so that

$$\Delta p_t^* = \Delta p_t + u_t$$

The measured real net investment series is the nominal series divided by the measured price:

$$
\begin{aligned}
\Delta k_t^* &= \Delta k_t + \Delta p_t - \Delta p_t^* \\
&= \alpha + \beta \Delta y_t - \gamma \Delta p_t + \epsilon_t + \Delta p_t - \Delta p_t^* \\
&= \alpha + \beta \Delta y_t - \gamma \Delta p_t^* + (1 - \gamma)(\Delta p_t - \Delta p_t^*) + \epsilon_t \\
&= \alpha + \beta \Delta y_t - \gamma \Delta p_t^* - (1 - \gamma) u_t + \epsilon_t
\end{aligned}
$$

Note now that

$$Cov\left(-\Delta p_t^*, -(1 - \gamma) u_t\right) = (1 - \gamma) \sigma_u^2$$

Thus, the sign of the bias in the estimate of γ depends on the value of γ itself. If $\gamma < 1$, then the bias is positive, while if $\gamma > 1$ the bias is negative. Since our estimate of the coefficient on the relative price of computing equipment is greater than one in magnitude, this suggests that, if measurement error is a factor, then the true coefficient is greater in magnitude than our estimate.[15]

5.3 Out-of-Sample Forecasting

Our interpretation of the results in Table 1 is that they are broadly consistent with the theoretical approach outlined in the previous section. However, what of the fact that prompted this exploration, the investment boom of the 1990s? To test whether our two-equation procedure for predicting net investment helps to explain the recent behavior of the capital stock, we estimated our preferred equations for computing and non-computing equipment (Columns 5 and 6 of Table 1) through 1989:4. We then simulated them out

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[15] In any case, we believe the evidence on NIPA price deflators suggests a sanguine interpretation of the measurement error problem. Recent research by Doms (1999) has shown that price declines measured from matched models (following the price of the same machine over time) are similar to the NIPA measures based on hedonic regressions.

of sample, taking the realized paths of output and the cost of capital as given, to obtain simulated capital stock series for computing and non-computing equipment.

Applying chain aggregation to our two simulated capital stock series, we obtained a simulated series for the aggregate capital stock. As shown in Figure 5, the two-equation system produces a series (the dotted line) that tracks the actual behavior of the equipment capital stock (the solid line) in the 1990s much better than the out-of-sample simulated series for the aggregate version of the same regression (the dashed line). The series generated by the aggregate regression, like the in-sample residuals from the aggregate net investment model in Figure 2, fall further and further behind observed capital stock growth as the 1990s proceed. In contrast, while the disaggregated system underpredicts actual capital stock growth somewhat for a number of periods from 1993 on, it moves back in line by the end of our sample (1997:4). The reason for the superior tracking performance of the disaggregated system is intuitive: This approach allows the massive decline in computing prices to feed through to capital accumulation far more than aggregate econometric regressions.

More important than the system's ability to track the aggregate capital stock, however, is its ability to explain the behavior of gross equipment investment. As the perpetual inventory depreciation rates for computing and non-computing are relatively stable over our sample, we can use a simple out-of-sample forecasting procedure for gross investment: We convert the disaggregated out-of-sample forecasts for capital stocks into forecasts for gross investment using the most recently observed depreciation rates. Applying this procedure to our system estimated through 1989:4 produces gross investment series for computing and non-computing investment. Aggregating these series, we obtain a good description of the recent behavior of aggregate equipment investment: Our simulated out-of-sample series for aggregate gross investment grows 6.9 percent per year over 1990-97, pretty close to the observed value of 7.5 percent. Moreover, as shown in Figure 6, our simulated series (the dotted line) captures the move to rapid investment growth in 1992 and the sustained high rate of growth thereafter. In contrast, an aggregate model—using the same specification and the 1989 aggregate depreciation rate—would have averaged about 3.1 percentage points too low over the period 1990-97 (the dashed line).

6 Conclusions

Boosted by exploding investment in computing equipment, the behavior of equipment investment in the U.S. in the 1990s has been unprecedented. Thus, it should not be too surprising that the traditional econometric models of investment, based as they are on historical correlations, have completely failed to explain the boom. We conclude that these developments provide three important lessons for macroeconomists:

- *Prices Matter*: Many previous studies have found limited roles for price variables, stressing the ability of an accelerator model to explain the cyclical behavior of investment. In contrast, we find an important role for equipment prices. Specifically, falling computer prices played a crucial role in the investment boom of the 1990s.

- *Depreciation Matters*: Most empirical studies have tended to ignore the role played by the replacement of depreciated capital. We have shown that an increasing depreciation rate was of first-order importance in the extraordinary behavior of equipment investment in the 1990s. Moreover, we have pointed to an important issue in the measurement of depreciation rates: Methodological changes to the NIPAs have made the standard measure of the average depreciation rate based on aggregate data invalid.

- *Aggregation Matters*: Depreciation rates vary widely across different types of equipment. Also, a model with rational expectations and adjustment costs tells us that the effects of cost of capital shocks will not be uniform across all types of equipment. We show that a two-equation system for net and gross investment in computing and non-computing equipment, estimated through 1989, is capable of explaining the magnitude and pattern of the U.S. equipment investment boom of the 1990s, while aggregate models completely fail.

Put simply, our explanation of equipment investment in the 1990s is that declining computer prices had a very large effect in boosting the accumulation of computer capital. Consequently, this led to even greater rates of replacement investment. Ultimately, of course, the true test of any model is its ability to forecast future developments. We hope that the future does not turn out to be as unkind to our empirical approach as the 1990s proved to be to the traditional econometric models.

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Appendices

A   The Data

Our dataset consists of quarterly series over 1950:1-1997:4 for real output, as well as real investment, real capital stock, and the cost of capital for total equipment, computing equipment, and non-computing equipment. Our output series is real 1992 dollar output of the private business sector, which is defined as GDP minus output from government and non-profit institutions and the imputed income from owner-occupied housing.

Our series on real investment for total equipment is private nonresidential producers' durable equipment expenditures from National Income and Product Accounts (NIPA) Table 5.5. The data for real computer expenditures is the Computers and Peripherals series from the same source. For real capital stock series for total equipment and computing equipment, we started with the annual NIPA capital stock data, which are available through 1997 and published in Department of Commerce (1998). These annual data, which represent year-end stocks, were then converted to quarterly series using an interpolation routine that sets the growth rate for each quarter according to its share in the annual total for investment expenditures.

Our series on real equipment investment and real capital stock excluding computing equipment were not created by subtracting the real series for computing equipment from the real aggregates. The lack of additivity of the chain-aggregation formula means that this is an incorrect calculation. Rather, in theory, we need to construct a new aggregate from the 26 disaggregated non-computing equipment categories. In practice, a "chain-subtraction" procedure which applies equation (9) to aggregate equipment and the negative for computer investment works just as well and does not require data on 26 investment series.

The cost of capital is measured using the Hall-Jorgenson rental rate formula:

$$C_t = P_t \left(R_t + \delta - \frac{\dot{P}_t}{P_t} \right) \left(\frac{1 - ITC - \tau * DEP}{1 - \tau} \right)$$

where P_t is the price of capital relative to the price of output, R_t is the real interest rate, ITC is the investment tax credit, DEP is the present value of depreciation allowances, and τ is the marginal corporate income tax rate. The relative price series, P_t, are defined relative

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to the deflator for private business output. The "capital gains" term is implemented as a three-year moving average of the percentage change in P_t. To construct the real interest rate, R_t, we subtracted expected inflation – proxied by the average inflation rate of the private business output deflator over the previous five years - from the nominal rate on Baa corporate bonds. We then added a constant "risk premium" that normalized this required rate of return so that its average equalled the average rate of return on physical capital in our sample (6.8 percent), where this is measured as the ratio of nominal capital income to the nominal capital stock. The tax term was constructed using data on investment tax credits and service lives (used in the calculation of depreciation allowances) from Gravelle (1994). We used $\delta = 0.31$ for computing equipment, $\delta = 0.13$ for non-computing equipment, and a nominal-capital-stock weighted average of these two rates for aggregate PDE. We use a nominal capital stock weighted average because of the problems with aggregate perpetual inventory depreciation rates discussed in Section 3.

B   Depreciation Rates with Chained Aggregates

What will the calculated aggregate depreciation rate from equation (8) look like with chain-aggregated data? To keep the analysis transparent, we will look at a simple case. The Fisher chain formula is somewhat cumbersome, so instead we will use the Tornqvist aggregation formula. This procedure weights the growth rate of each category according to its share in the nominal aggregate, and produces aggregates with almost identical properties to the Fisher procedure. We will also make the following assumptions. Both types of capital depreciate at the same rate δ; the price of type-A capital falls at rate γ relative to the price of type-B capital and output, which are both normalized to equal one. Finally, firms produce with a Cobb-Douglas production function ($Q_t = A_t \left(K_t^A \right)^\alpha \left(K_t^B \right)^{1-\alpha}$) and there are no adjustment costs. Now, assuming no taxes, the cost of capital for type A simplifies to $P^A(r + \delta + \gamma)$. The cost of capital for type B is $(r + \delta)$. Given our assumptions, firms accumulate capital according to the first-order conditions:

$$K_t^A = \frac{\alpha Q_t}{P_t^A(r + \delta + \gamma)}$$

$$K_t^B = \frac{(1 - \alpha)Q_t}{(r + \delta)}$$

These conditions imply capital stock growth rates $g^A = g^Q + \gamma$ and $g^B = g^Q$. Using the Tornqvist formula, the growth rate for the chain-aggregated capital stock is

$$g^{CW} = \theta(g^Q + \gamma) + (1 - \theta)g^Q = g^Q + \alpha\gamma$$

where θ is the share of capital of type A in the aggregate nominal capital stock. Note that these nominal stocks are defined as the "replacement value" of the capital stock and are obtained by reflating the real capital stock for each category by the current-period price of new capital. Now consider the behavior of a chain-aggregate for real investment. Re-arranging the expressions for the growth rate of the capital stock we get:

$$\frac{I_t^A}{K_{t-1}^A} = g^Q + \gamma + \delta$$

$$\frac{I_t^B}{K_{t-1}^B} = g^Q + \delta$$

Thus, for each type of capital, the ratio of real investment to the real capital stock is a constant. So, real investment for capital of types A and B also grow at rate g^A and g^B.

To calculate the growth rate of the chain aggregate, we need nominal shares of investment:

$$
\begin{aligned}
\frac{P_t^A I_t^A}{P_t^B I_t^B} &= \left(\frac{I_t^A}{K_{t-1}^A}\right)\left(\frac{K_{t-1}^B}{I_t^B}\right)\left(\frac{K_t^B}{K_{t-1}^B}\right)\left(\frac{K_{t-1}^A}{K_t^A}\right)\left(\frac{P_t^A K_t^A}{P_t^B K_t^B}\right) \\
&= \left(\frac{g^Q + \delta + \gamma}{g^Q + \delta}\right)\left(\frac{g^Q + 1}{g^Q + 1 + \gamma}\right)\left(\frac{P_t^A K_t^A}{P_t^B K_t^B}\right) \\
&> \left(\frac{P_t^A K_t^A}{P_t^B K_t^B}\right)
\end{aligned}
$$

The share of capital of type A in nominal *investment* is larger than its share in the nominal *capital stock*. The reason for this is intuitive. The real capital stock of type A is growing faster than the real stock of type B. This means that, measured in today's dollars at replacement cost, there is more investment relative to the capital stock for type A than there is for type B; as a result the nominal share of investment for type A is higher. Since real investment of type A grows at rate $g_Q + \gamma$ while real investment of type B grows at rate g_Q, the growth rate of the Tornqvist chain aggregate for real investment places more weight on the faster growing category than does the corresponding growth rate for the aggregate capital stock. Hence, the chain aggregate for investment will always grow faster

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than the chain aggregate for the capital stock. This example, in which relative price shifts cause the fast growing category to have a larger share in nominal investment than in the nominal capital stock, lines up precisely with reality: Computers currently have a much larger share in nominal equipment investment (14 percent in 1997) than in the nominal equipment capital stock (5 percent in 1997).

Now, suppose we solve for the aggregate depreciation rate from the chain-aggregates for investment and the capital stock:

$$\delta_t^{CW} = \frac{I_t^{CW}}{K_{t-1}^{CW}} - g_t^{CW}$$

Then this value will equal δ only in the base year. Since I^{CW} grows faster than K^{CW} in each period, this "depreciation rate" gets larger each period. More generally, if we allowed the two types of capital to have varying depreciation rates, the depreciation rate estimated from this equation would only equal a weighted average of the underlying depreciation rates in the base year, as we move forward from the base year this measure would eventually be higher than each of the underlying depreciation rates.

C Omitted Proof

Proof that $\mu(1)(1 - \theta\lambda) = 1$ when $\pi(1) = 1$:

Inserting the expression for $\mu(1)$ we need

$$(1 - \theta\lambda)\left[1 + \sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}(\theta\lambda)^{r-k}\pi_r\right)\right] = 1 - \pi(\theta\lambda)$$

Re-arranging the left-hand-side of this equation we get

$$(1 - \theta\lambda)\left[1 + \sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}(\theta\lambda)^{r-k}\pi_r\right)\right] = (1 - \theta\lambda)\left[1 + \sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}\pi_r\right)(\theta\lambda)^k\right]$$

Now use $\pi(1) = 1$:

$$(1 - \theta\lambda)\left[1 + \sum_{k=1}^{m-1}\left(\sum_{r=k+1}^{m}(\theta\lambda)^{r-k}\pi_r\right)\right] = (1 - \theta\lambda)\left[1 + \sum_{k=1}^{m-1}\left(1 - \sum_{r=1}^{k}\pi_r\right)(\theta\lambda)^k\right]$$

Expanding this expression we get

$$1 + (1 - \pi_1)(\theta\lambda) + (1 - \pi_1 - \pi_2)(\theta\lambda)^2 + \ldots\ldots + (1 - \pi_1 - \pi_2 - \ldots - \pi_{n-1})(\theta\lambda)^{m-1}$$
$$-\theta\lambda - (1 - \pi_1)(\theta\lambda)^2 - (1 - \pi_1 - \pi_2)(\theta\lambda)^3 - \ldots\ldots\ldots - .(1 - \pi_1 - \pi_2 - \ldots - \pi_{n-1})(\theta\lambda)^m$$
$$= \quad 1 - \pi_1(\theta\lambda) - \pi_2(\theta\lambda)^2 - \ldots - \pi_n(\theta\lambda)^n$$
$$= \quad 1 - \pi(\theta\lambda)$$

as required.

Table 1

Capital Stock Growth Regressions

(Standard errors in parentheses)

	Total	Computers	Excluding Computers	Total	Computers	Excluding Computers
	(50-97)	(80-97)	(50-97)	(50-97)	(80-97)	(50-97)
λ	0.93	0.90	0.92	0.94	0.86	0.93
	(.02)	(.04)	(.02)	(.02)	(.05)	(.02)
Sum of the Coefficients on:						
Output	0.10	0.18	0.11	0.10	0.21	0.10
	(.01)	(.13)	(.02)	(.01)	(.10)	(.02)
Cost of capital	-0.02	-0.17	-0.01			
	(.01)	(.09)	(.005)			
Relative prices				-0.03	-0.26	-0.03
				(.01)	(.10)	(.02)
Cost of capital without prices				-0.01	-0.08	-0.01
				(.005)	(.08)	(.006)
Long-run Elasticities:						
Output	1.49	1.75	1.33	1.46	1.44	1.33
	(.36)	(1.41)	(.31)	(0.37)	(0.84)	(.32)
Cost of capital	-0.18	-1.59	-0.13			
	(.09)	(.75)	(.083)			
Relative price of capital				-0.45	-1.83	-0.33
				(.26)	(.45)	(.31)
Cost of capital without relative price				-0.18	-0.53	-0.14
				(.11)	(.50)	(.10)

Figure 1
The Demise of the Traditional Investment Regression

Investment relative to the Capital Stock
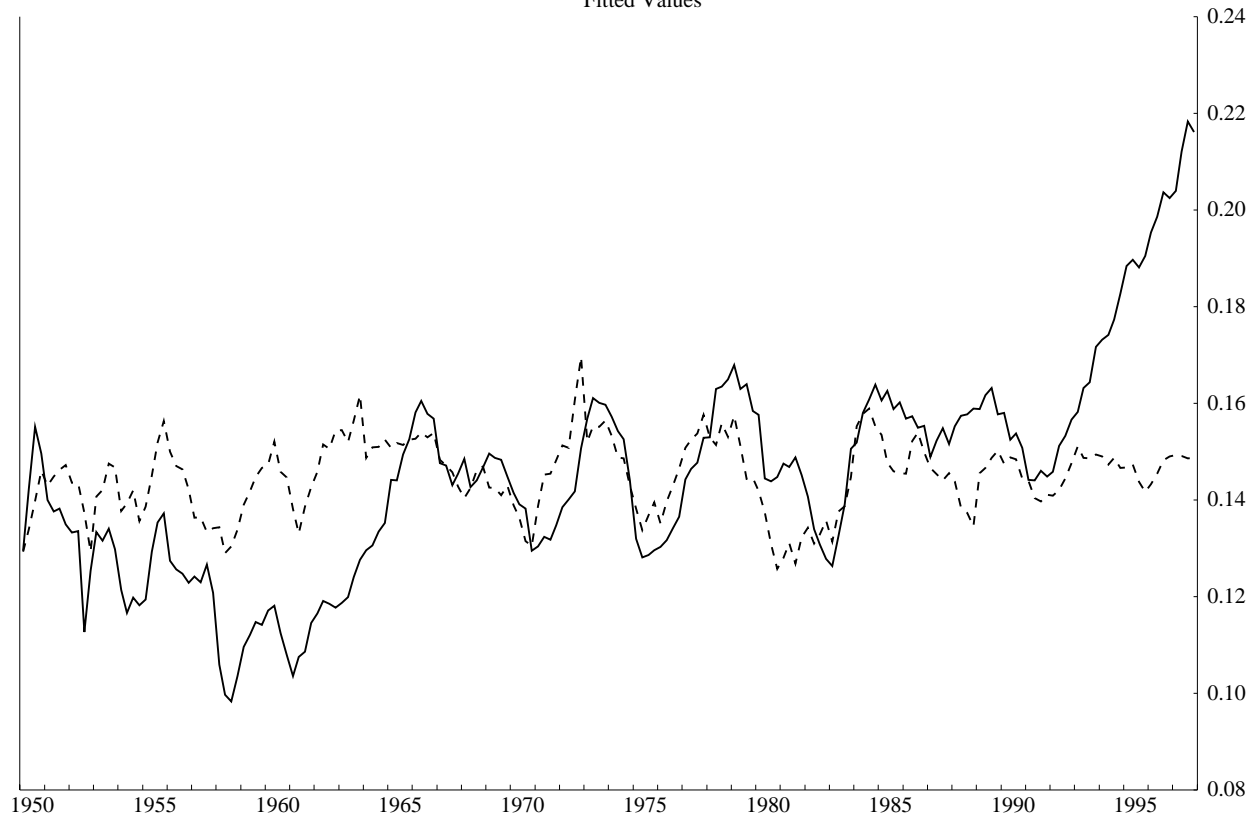
——— Data
- - - Fitted Values

Figure 2
Perpetual Inventory Depreciation Rates for Aggregate Equipment

——— Using Chain-Weight Investment and Capital
– – – Using Fixed-Weight Investment and Capital
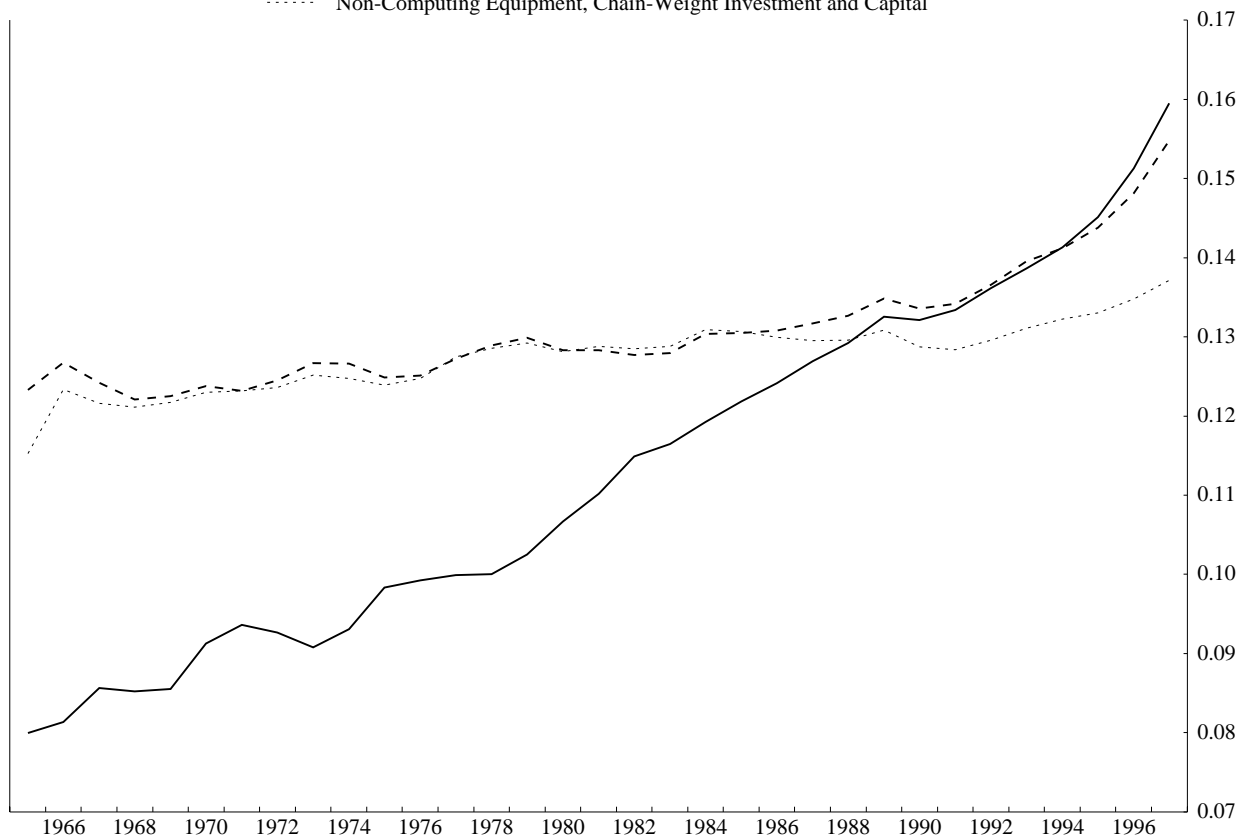··········· Non-Computing Equipment, Chain-Weight Investment and Capital

Figure 3
Residuals from Gross and Net Aggregate Equipment Investment Regressions
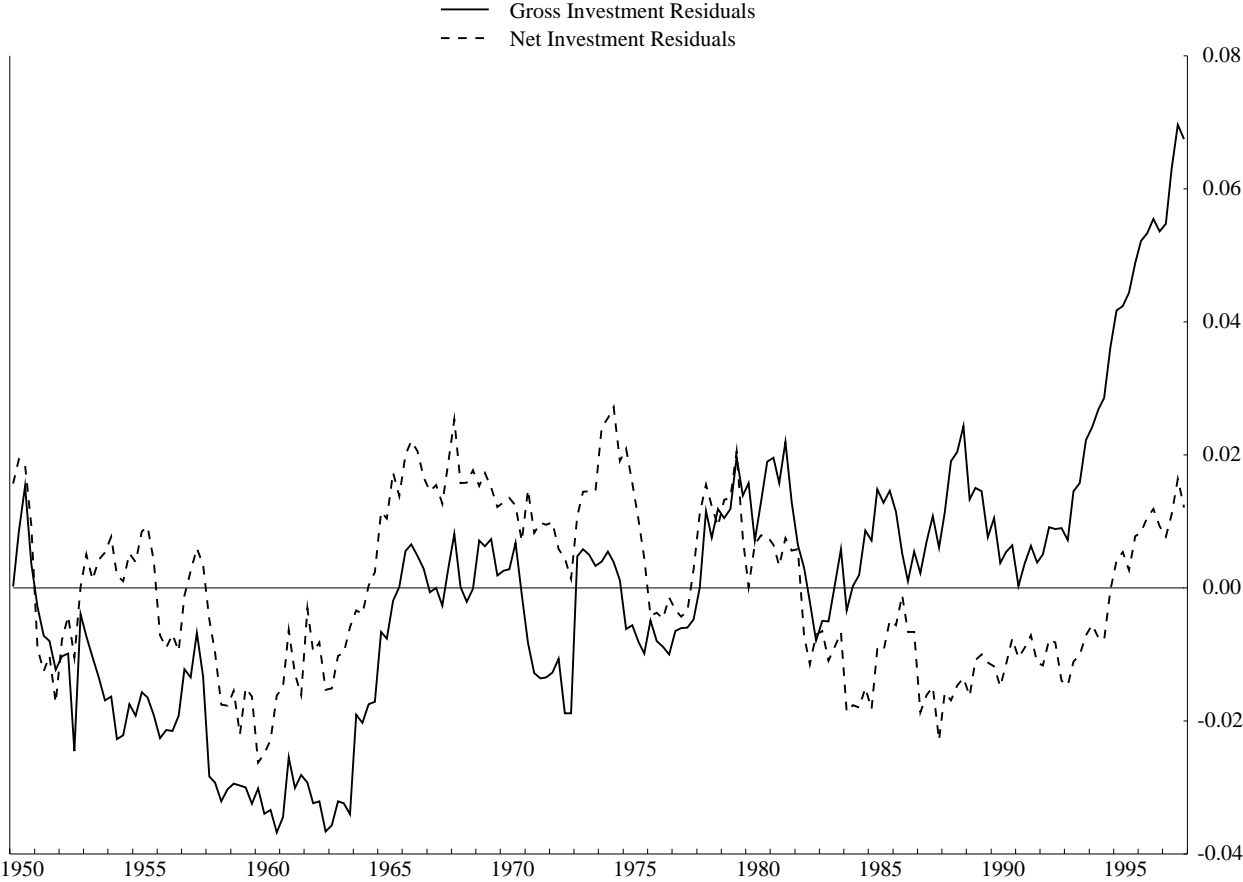Errors Expressed As Percentages of the Capital Stock
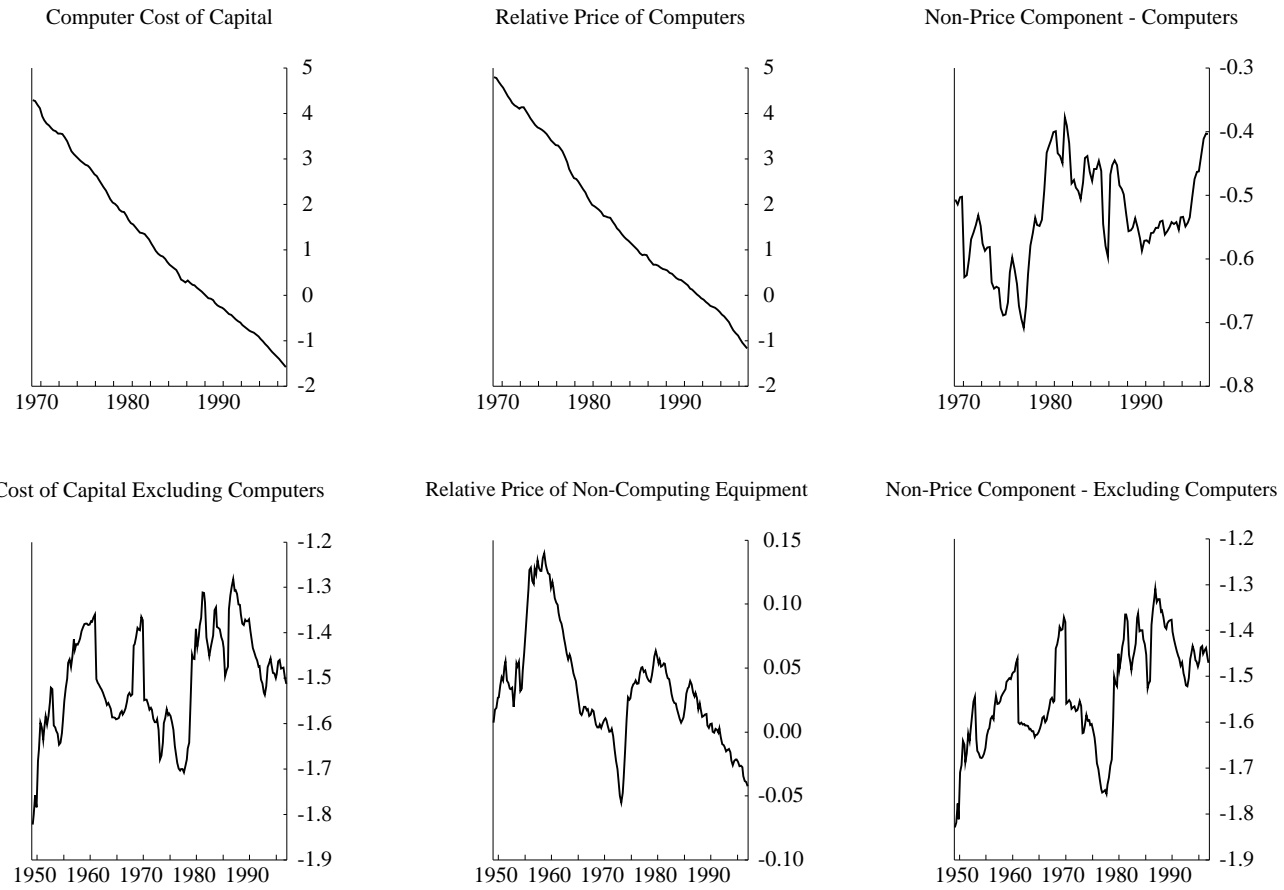
Figure 4
The Cost of Capital
In Logs

Computer Cost of Capital

Relative Price of Computers

Non-Price Component - Computers

Cost of Capital Excluding Computers

Relative Price of Non-Computing Equipment

Non-Price Component - Excluding Computers

Figure 5
Out-of-Sample (Post-1989) Forecasts of Growth in the Stock of Equipment
Annualized Growth Rates

———— Data
- - - - Aggregate Model (Accelerator with Cost of Capital)
.... Disaggregated Model (Same Specification)
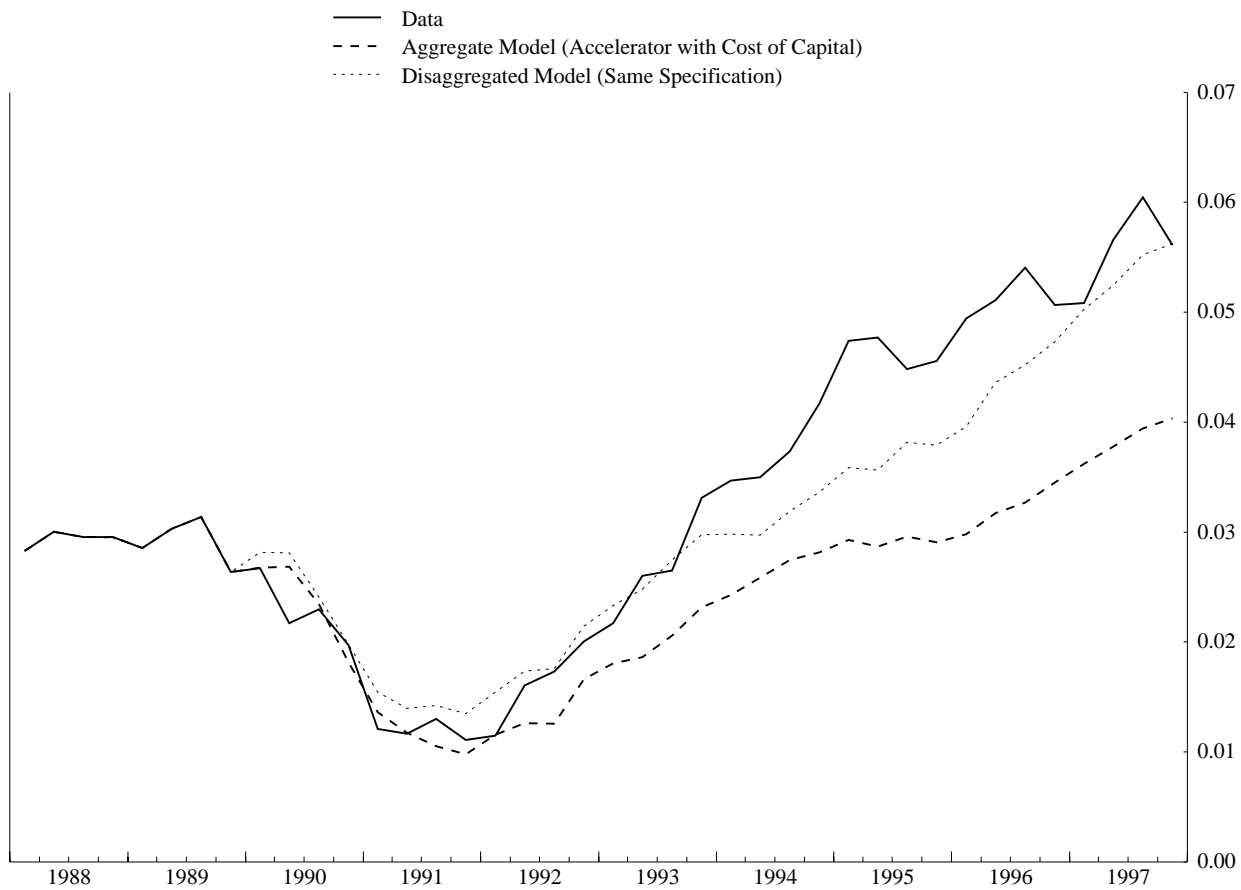
Figure 6
Out-of-Sample (Post-1989) Forecasts of Growth in Real Equipment Investment
Year-over-Year Percentage Changes